Directivity of shear waves in thermoelastic regime

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- Context and aim
- Methodology
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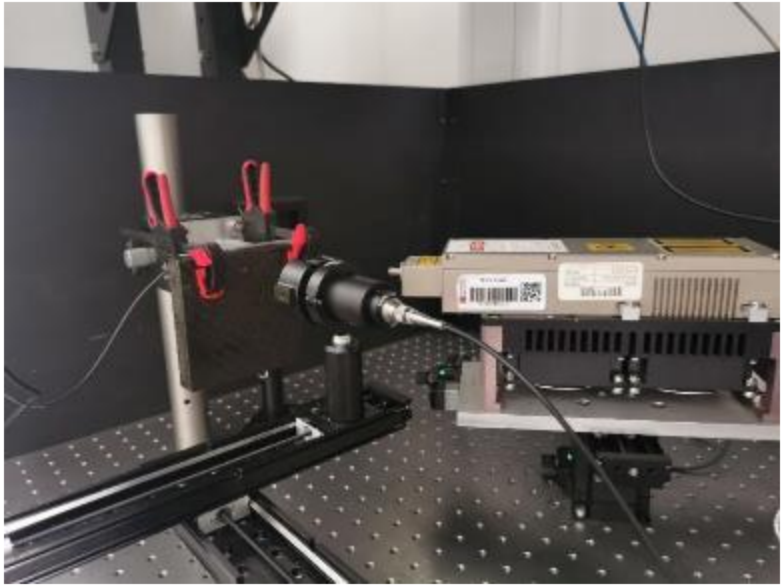
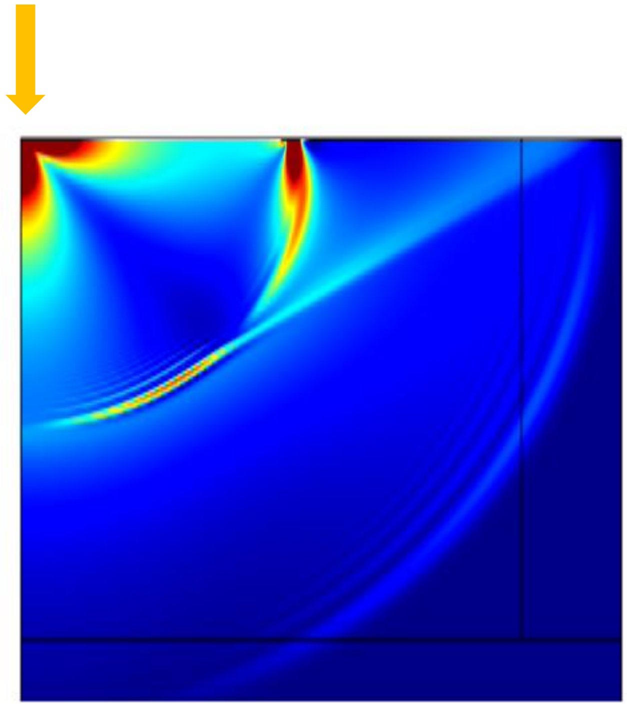


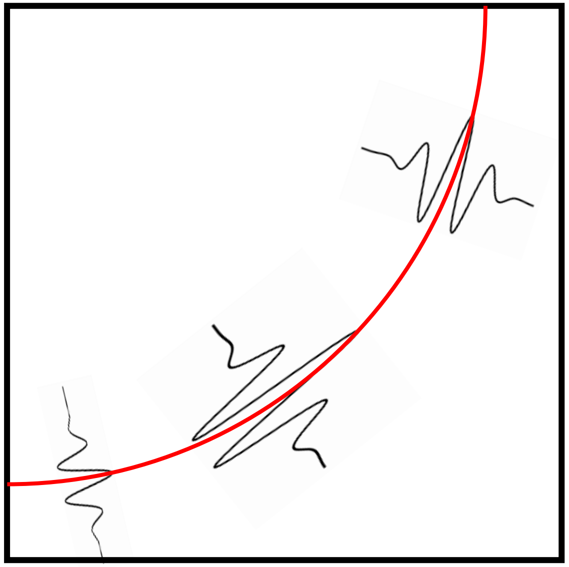
Context and aim

- Laser induced ultrasonic waves in thermoelastic regime
- Shear waves commonly used for defect detection



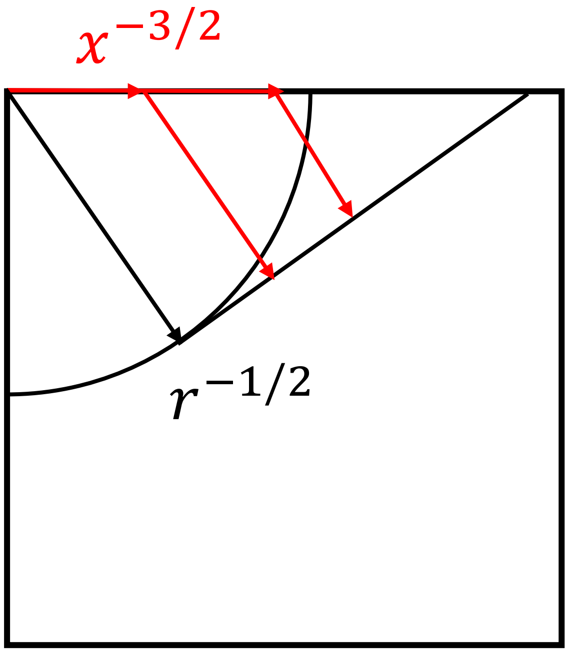
Context and aim

- Directivity of shear waves
- Angle-dependent amplitude
- Indicate the strength of the signal at different angles



Context and aim

- Head wave interference
- Same velocity as shear wave inside material
- Different wavefront and decay rate from shear wave [1]

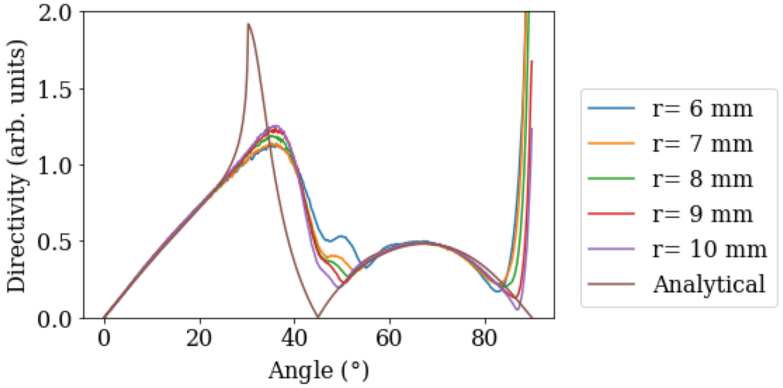


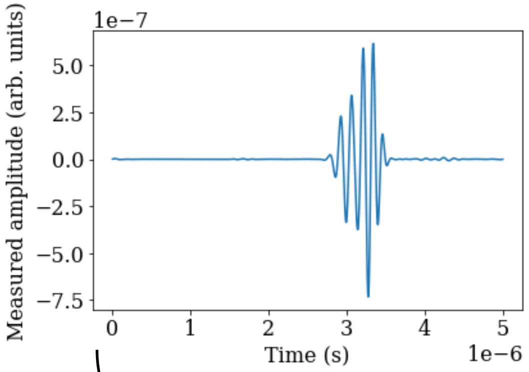
x: horizontal distance from centre
r: distance from centre

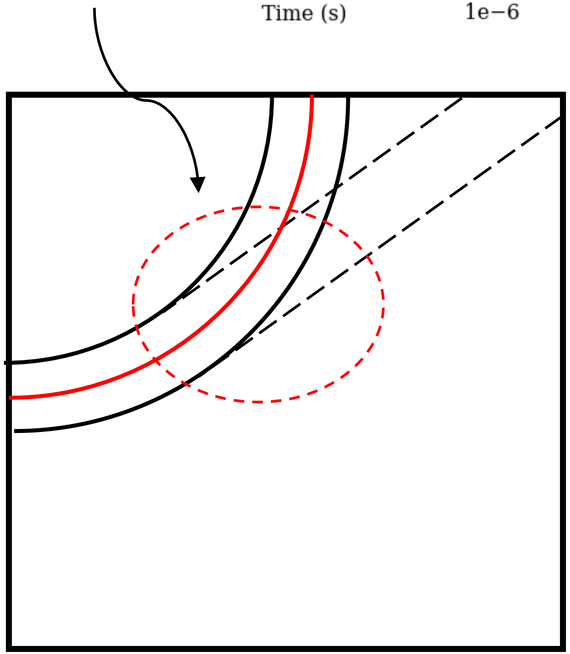
[1] Fradkin, Larissa Ju, and Aleksei P. Kiselev. "The two-component representation of time-harmonic elastic body waves in the high-and intermediate-frequency regimes." *The Journal of the Acoustical Society of America 101*, no. 1 (1997): 52-65.

Context and aim

- Obtain a directivity independent of radial distances from source
- Without using a large simulation domain

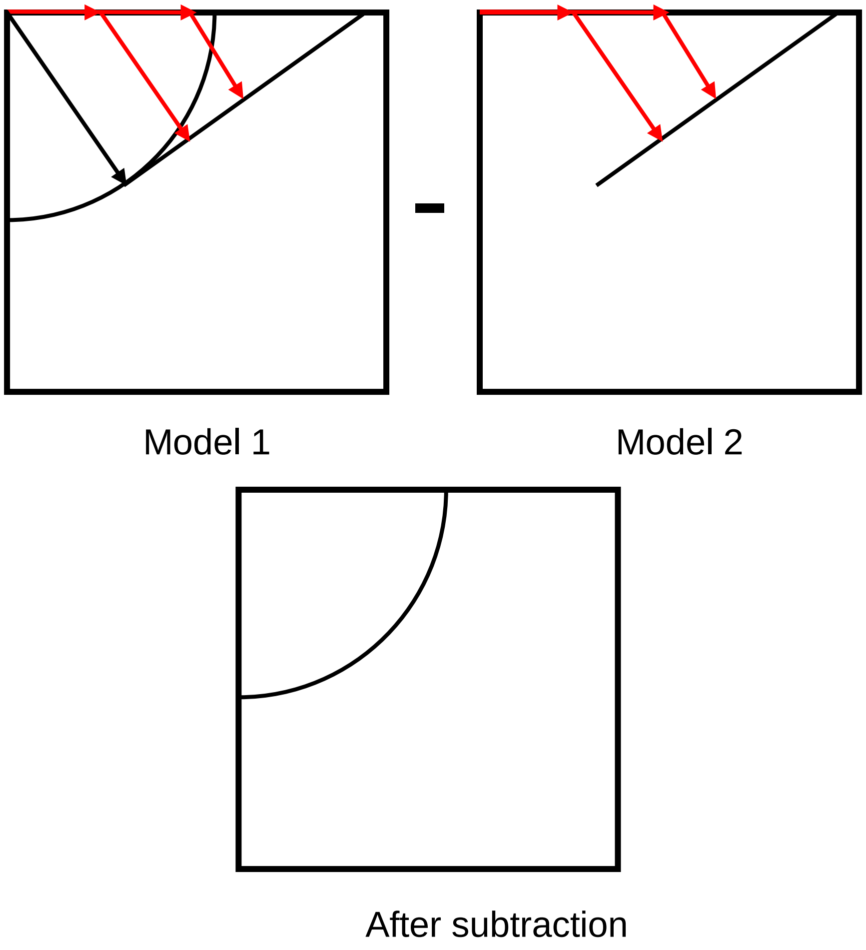






Method

- Simulation of ultrasound generation (Model 1)
- Simulate a head wave dominated wave field (Model 2)
- By suppressing the initial displacement contributing to shear waves
- Subtract head wave field from Model 1



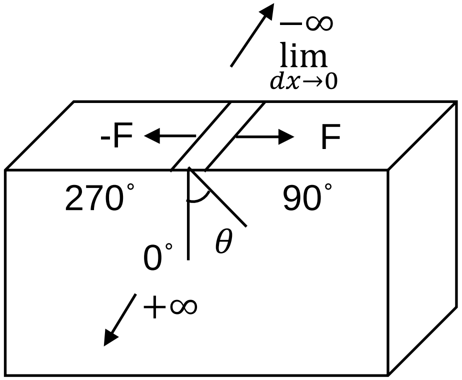
Method

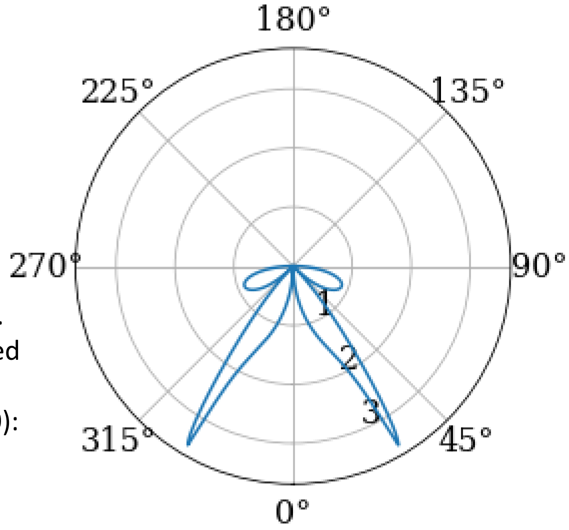
- Benchmark analytical solution [2]
- Assumptions:
  - Force dipole
  - Acting on surface
  - Isotropic, homogenous material
- Independent of source temporal profile and frequency content

$$G_T(\theta) \propto \frac{sin2\theta\ cos2\theta}{cos^2 2\theta + 2\ sin\theta\ sin2\theta(\kappa^{-2} - sin^2\theta)^{1/2}},$$
$$\kappa = c_L/c_T$$

θ: observation angle
c_L: velocity of longitudinal wave
c_T: velocity of shear wave

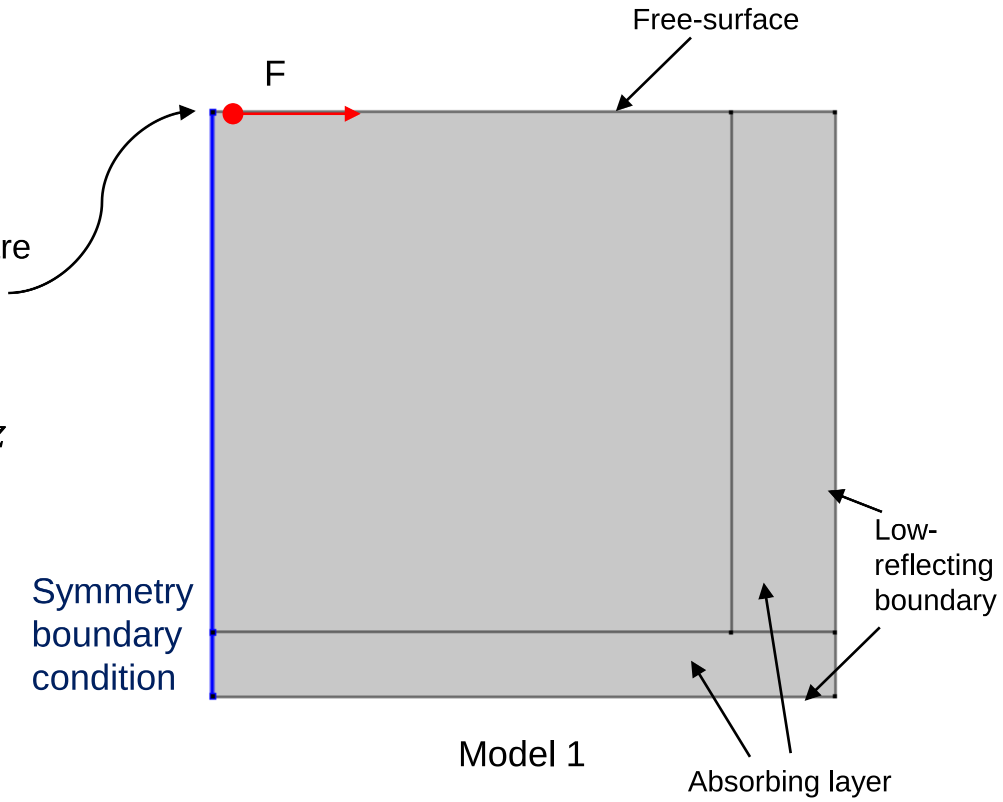
[2] Bernstein, Johanna R., and James B. Spicer. "Line source representation for laser-generated ultrasound in aluminum." The Journal of the Acoustical Society of America 107, no. 3 (2000): 1352-1357.





Method

- 2D domain ($10mm \times 10mm$), symmetrical about central axis
- Boundary conditions and loading:
  - **Model 1:** point load at nearest node to centre tangential to surface, broadband signal
  - **Model 2:** surface displacement taken from Model 1, displacement at centre damped
- Mesh: linear elements, 30 elements per shear wavelength at frequency of $10MHz$
- Time step: $5e-9s$
- Material: Aluminium, isotropic
  - Poisson's ratio: $v = 0.33$
  - Young's modulus: $E = 69GPa$



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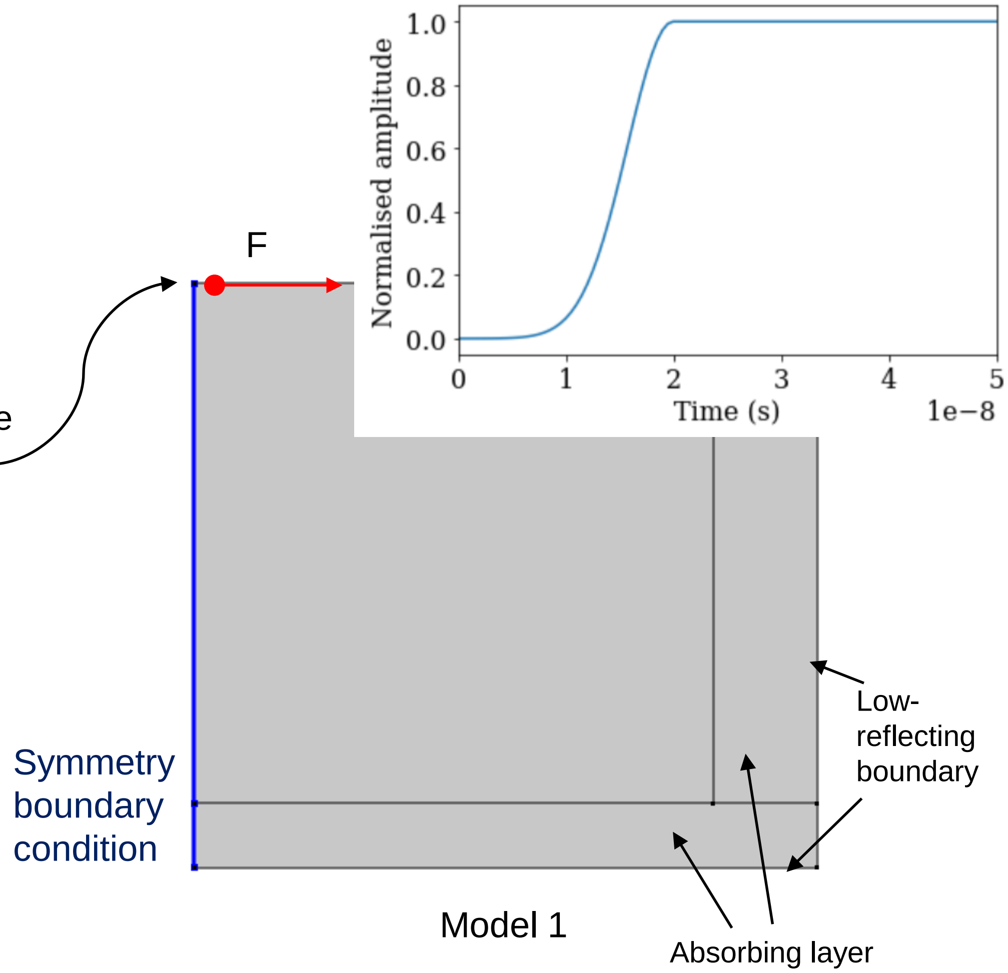

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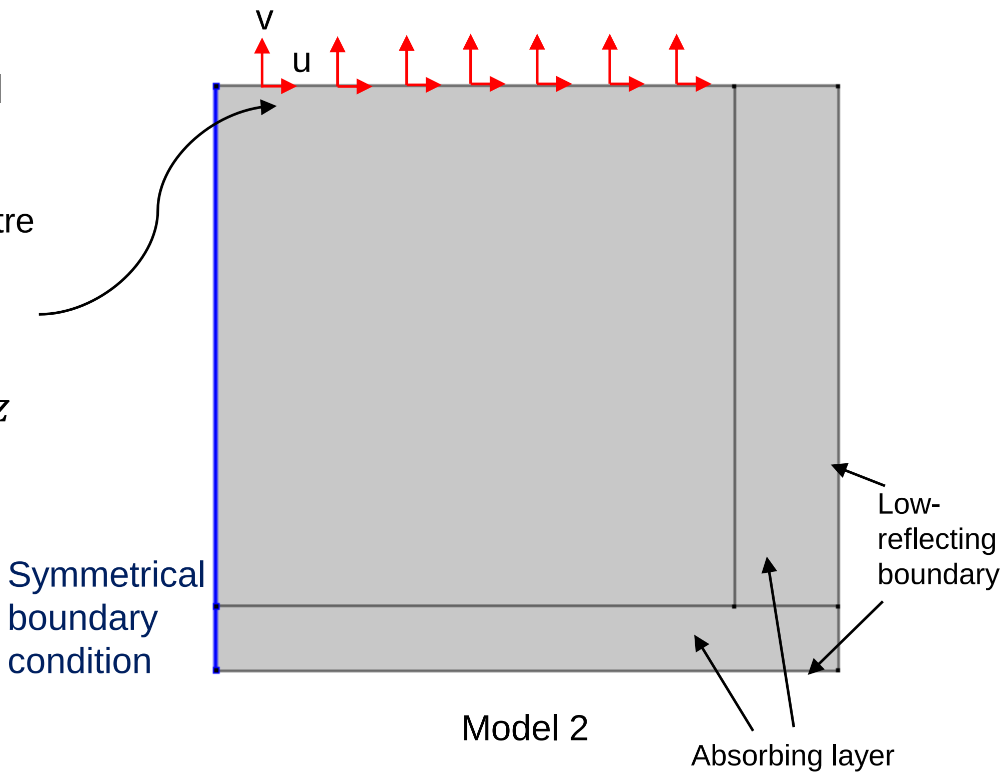


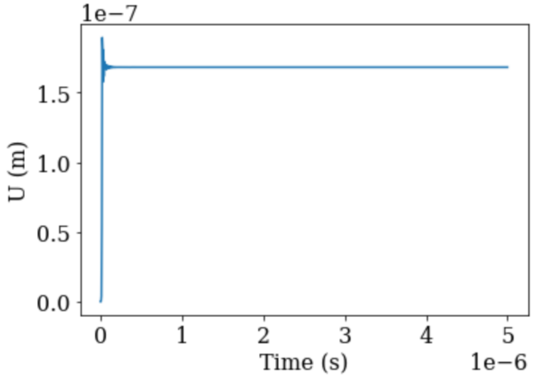
Model 2

Method

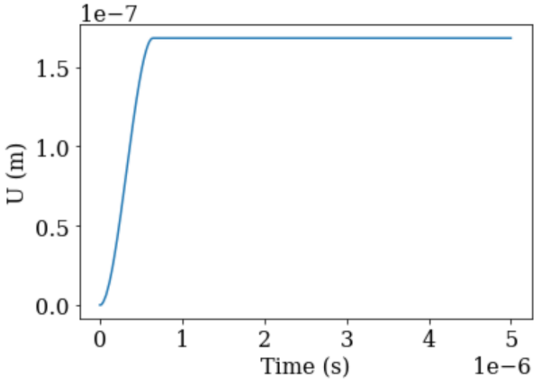
- Apply temporal smooth step to damp initial displacement
- $$S_1(x) = \begin{cases} 0 & x \le 0 \\ 3x^2 - 2x^3 & 0 \le x \le 1 \\ 1 & 1 \le x \end{cases},$$
  $$x = t/\tau$$
- The first derivative is continuous

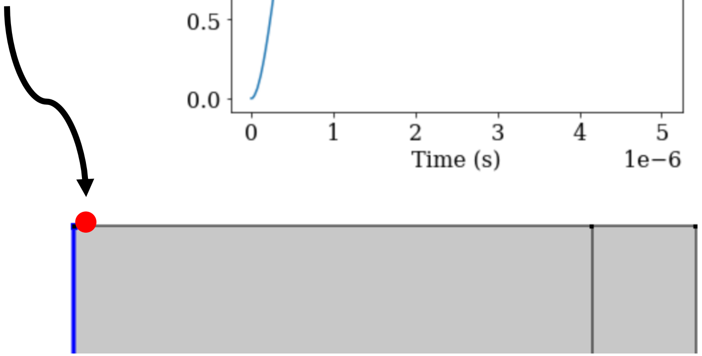
x: adjustable parameter of the smooth step
t: observation time
τ: duration of frequency filtered shear wavepacket

original



damped





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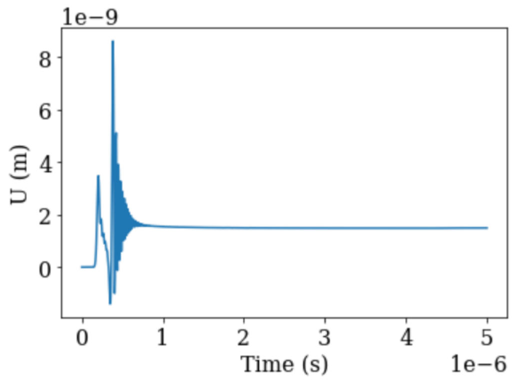

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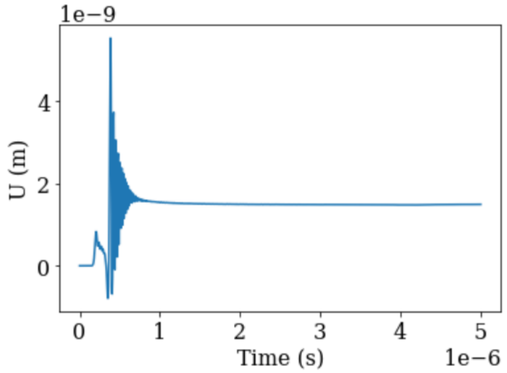

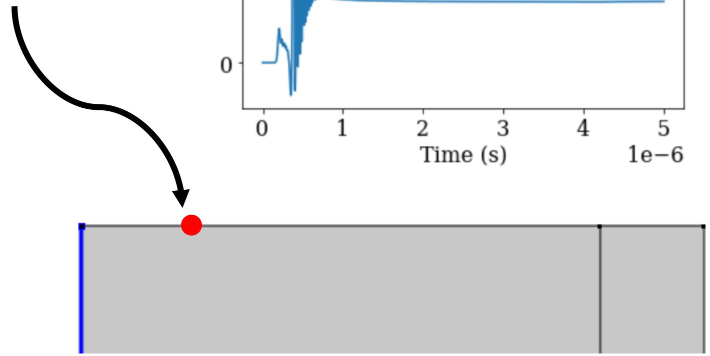

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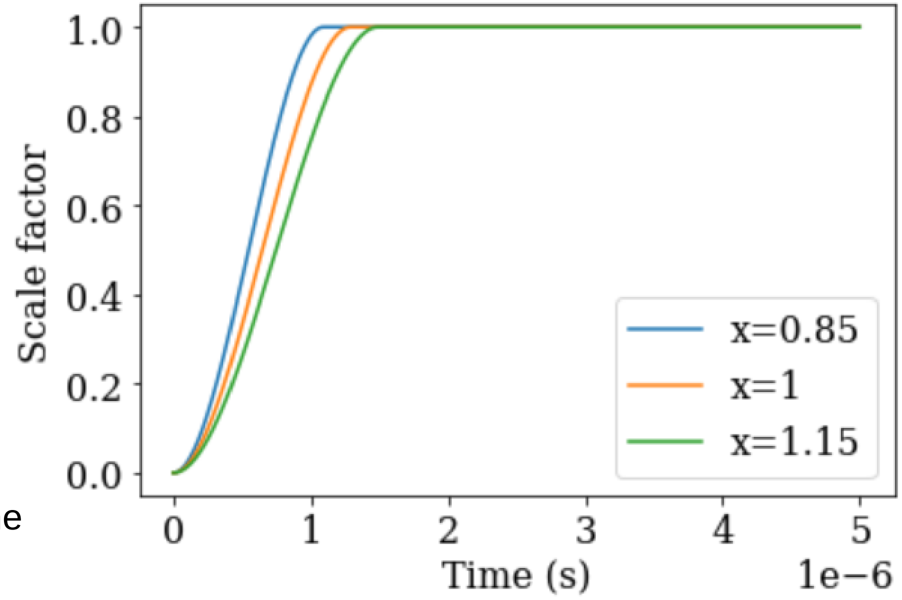
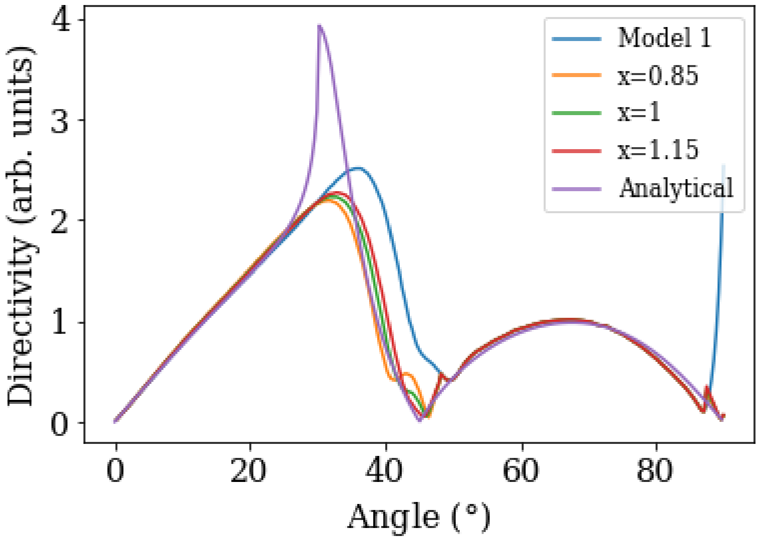


Method

- Determine the length of smooth step
- The duration of generated shear wave is $\tau = 6.67e-7s$
- Ideally the smooth step would cover the entire duration
- $x = t/\tau = 1$ fits best to analytical solution



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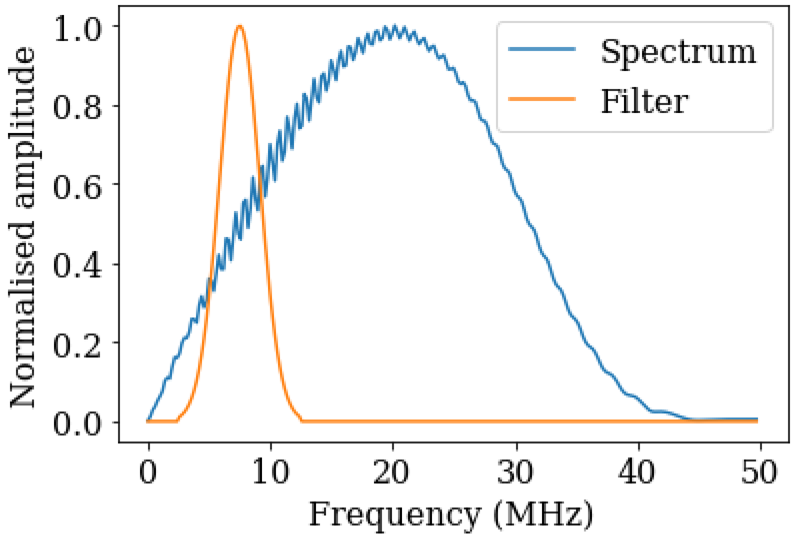
Method

- Extract waves from displacements
  - curl of displacement, rotational motion

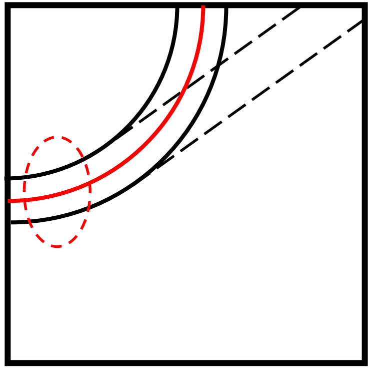
$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

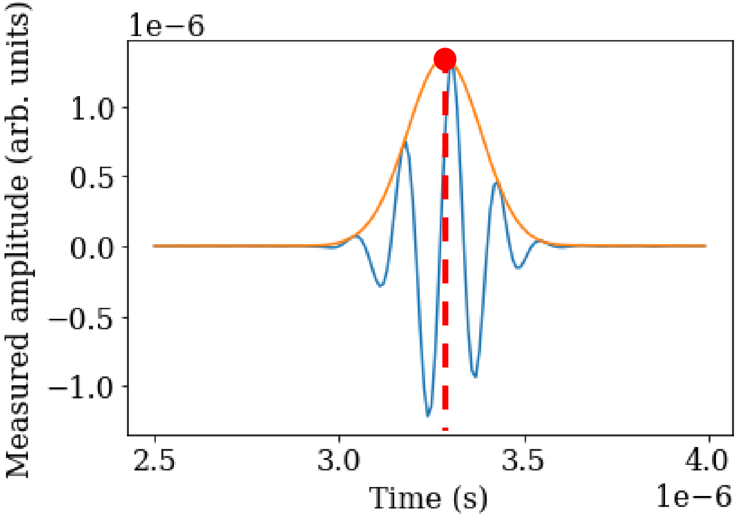
- Gaussian frequency filter
  - Centre frequency = 7.5MHz
  - Bandwidth = 5MHz



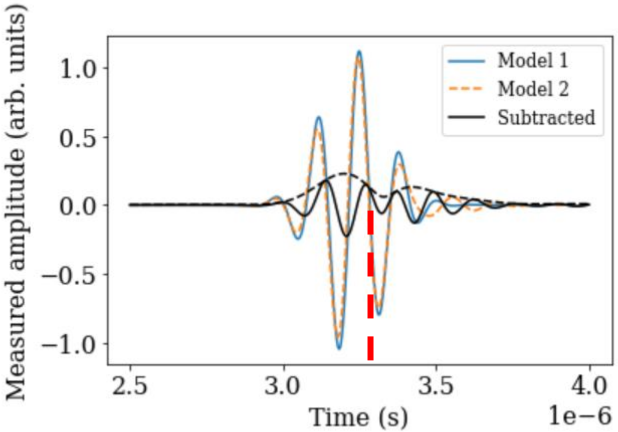
Method

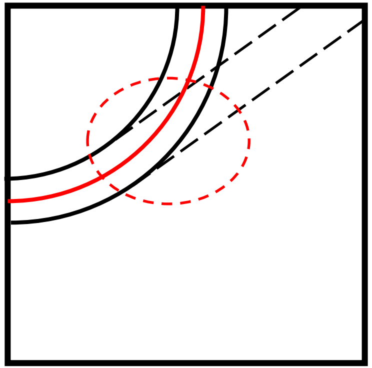
- Subtraction
- Obtain directivity
  - Determine shear wave arrival time from signals at head-wave-free angles
  - Extract amplitude of the signal envelope at this time instance



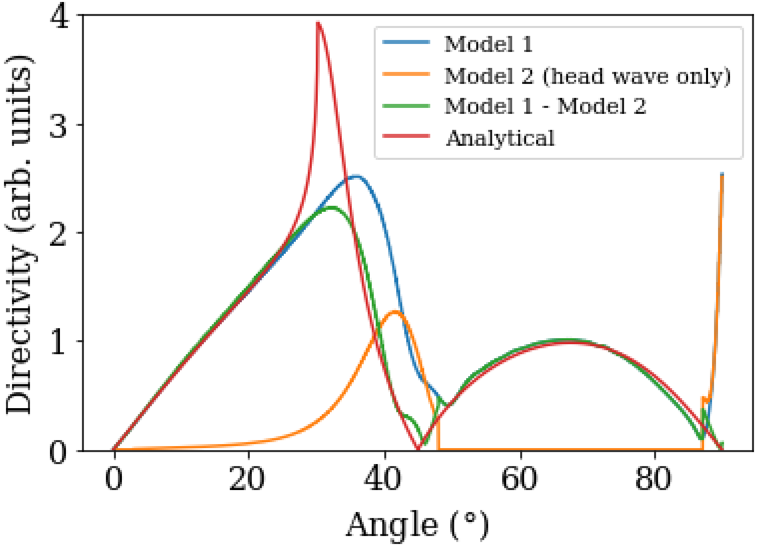


Results and discussions





- Agrees well with the analytical directivity
- Except 25˚- 30˚
- Due to finite separation between point loads, residual shear waves, etc.



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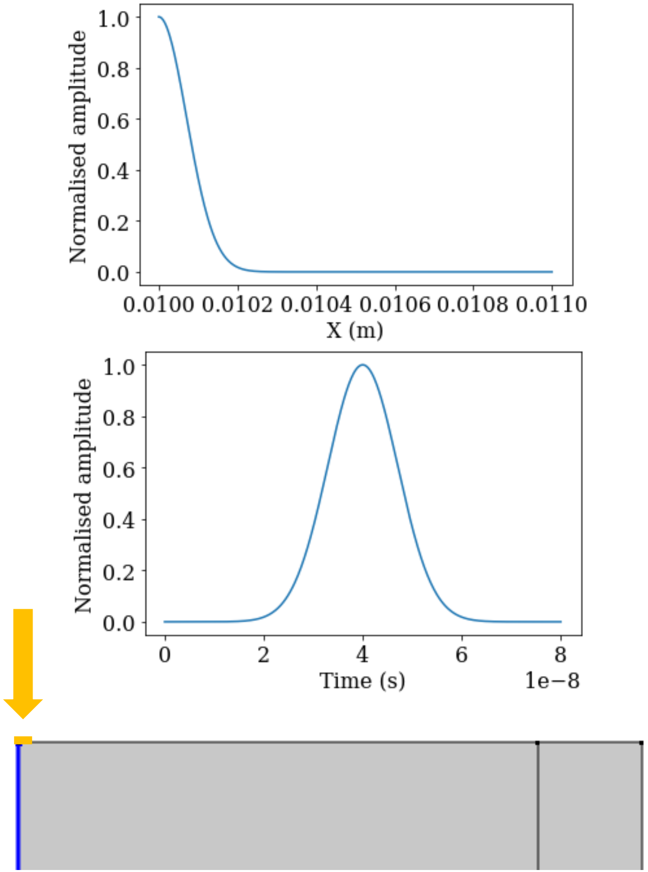
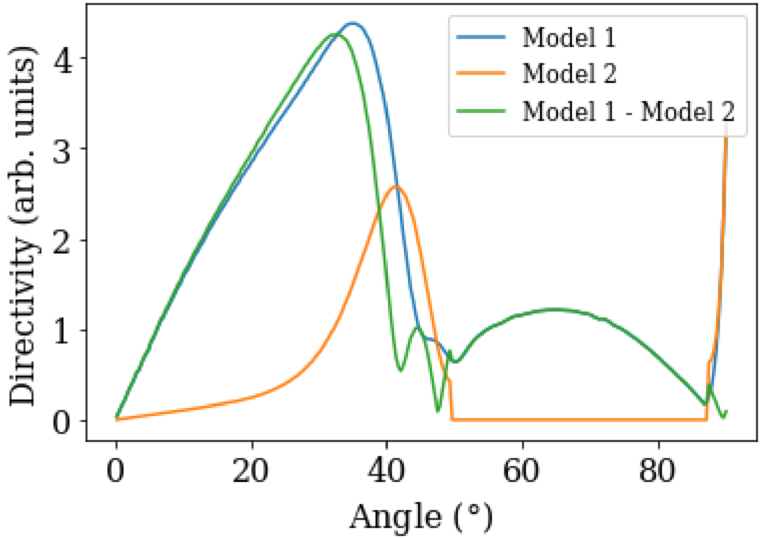
Results and discussions

- More realistic simulation
- Coupling heat transfer and solid mechanics
- The centre of expansion locates at a small distance below surface
- Boundary condition and loading:
  - Inward heat flux
  - Gaussian profile in space and time
  - Spot size of $0.2mm$
  - Pulse duration $20ns$
  - Free surface



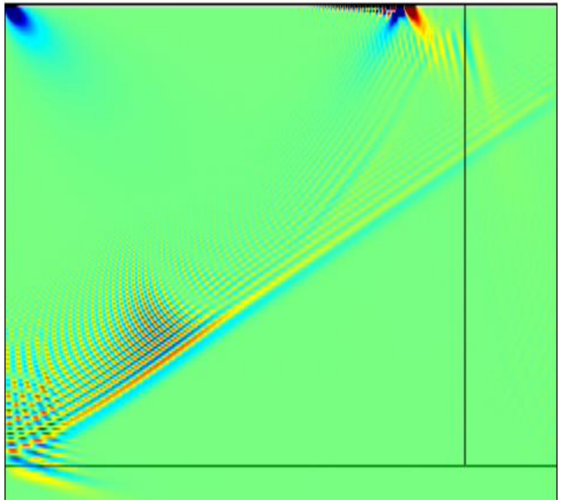
Results and discussions

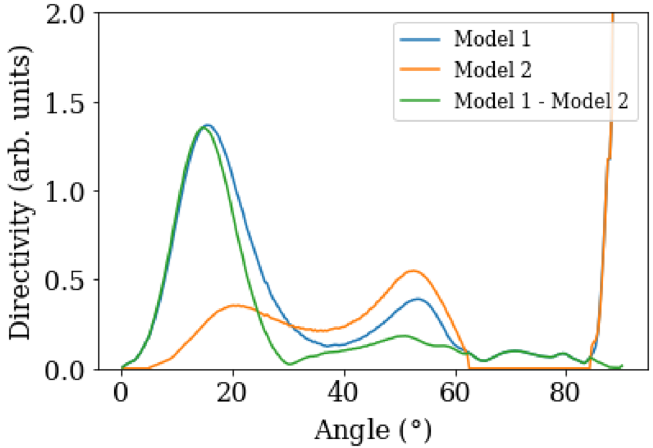
- More realistic simulation
- Coupling heat transfer and solid mechanics
- The centre of expansion locates at a small distance below surface
- Head wave interference 40˚- 50˚



Results and discussions

- Austenitic welds
- Material:
  - Steel, transversely isotropic
  - Elasticity matrix
  - $$\begin{pmatrix} 241.1 & 96.92 & 138.03 & 0 & 0 & 0 \\ 96.92 & 241.1 & 138.03 & 0 & 0 & 0 \\ 138.03 & 138.03 & 240.12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 112.29 & 0 & 0 \\ 0 & 0 & 0 & 0 & 112.29 & 0 \\ 0 & 0 & 0 & 0 & 0 & 72.09 \end{pmatrix} GPa$$
- Require more work to determine the shear wave arrival time, separation of quasi- longitudinal and shear waves, etc.
- Need to rethink the definition of directivity





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Summary

- The head wave interferes with shear wave, causing the directivity measurement to vary at different radial positions
- A method of subtracting head wave is used to obtain a directivity independent of radial positions without having to run simulations in big domains

Future work

- Experimental validation
- Application to composites

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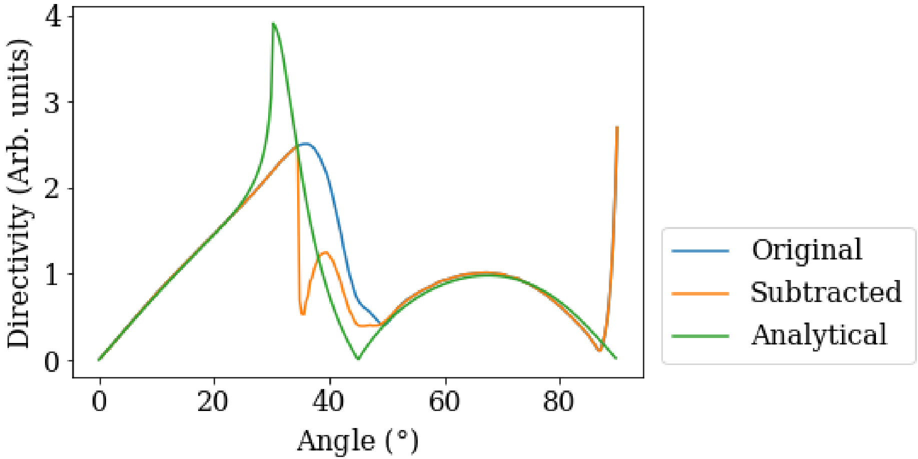
Appendix

- Backpropagation
  - Use the knowledge of head wave ray path and velocity to calculate arrival time
  - Use a known waveform to subtract from the signal
- Phase shift
- Amplitude
  - Analytical model suggests attenuation of $x^{-3/2}$, but in simulation it does not fit exactly when approaching critical region



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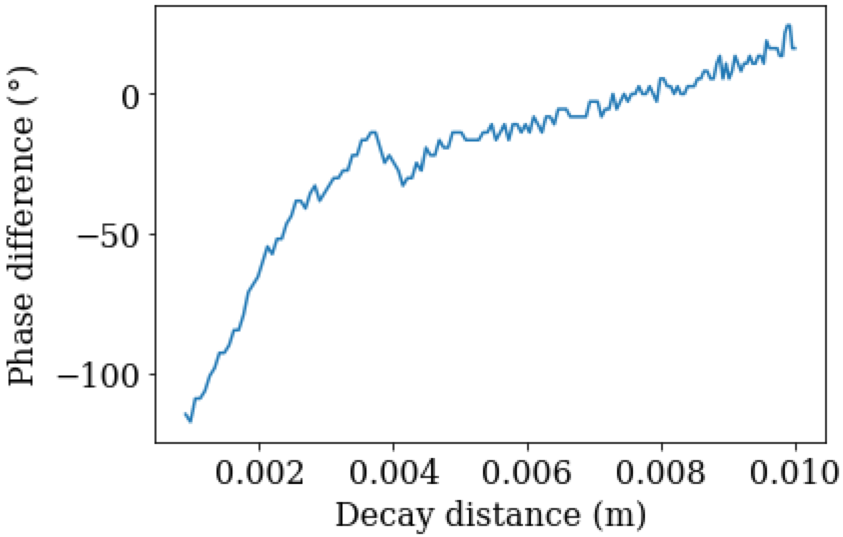

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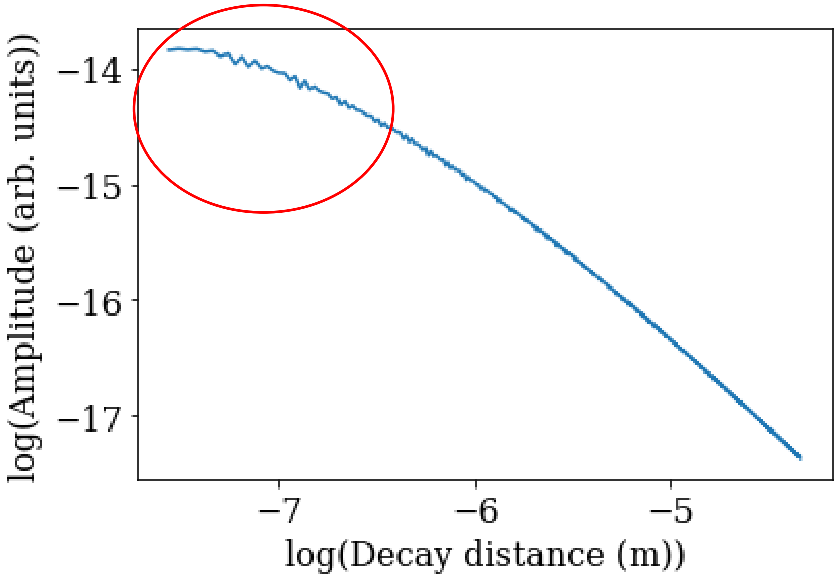
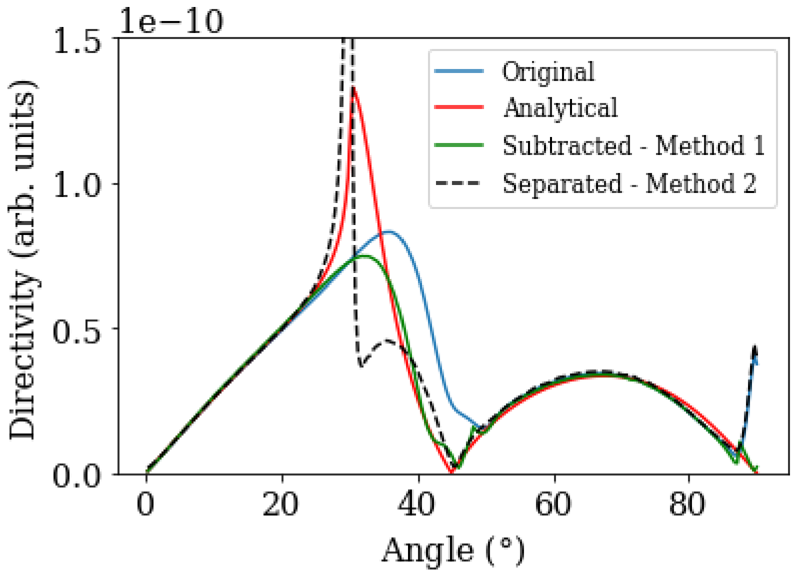


Appendix

- Separation by angle
- Shear wave has circular wavefront
- Head wave propagates in one direction – plane wave
- Solve two simultaneous equations to obtain the contribution of shear and head wave in u and v
- Less stable in the critical region than subtraction



Appendix

- Derivation

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda + \mu)\nabla(\nabla \mathbf{u}) + \mu \nabla^2 \mathbf{u}$$

Given $\nabla^2 \mathbf{u} = \nabla(\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u})$, so

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda + 2\mu)\nabla(\nabla \mathbf{u}) - \mu \nabla(\times \nabla \times \mathbf{u})$$

Let $c_l = \sqrt{\frac{\lambda+2\mu}{\rho}}$, $c_s = \sqrt{\frac{\mu}{\rho}}$,

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = c_l^2 \nabla(div\mathbf{u}) - c_s^2 \nabla \times curl\mathbf{u}$$

Take the divergence and curl of both sides gives

$$c_l^2 \nabla^2 (div\mathbf{u}) = \frac{\partial^2 div\mathbf{u}}{\partial t^2}$$

$$c_s^2 \nabla^2 (curl\mathbf{u}) = \frac{\partial^2 curl\mathbf{u}}{\partial t^2}$$

Appendix

- Calculate the size of simulation domain that the directivity extracted from original simulation has a head wave interference region as small as the subtracted one
- The directivity currently obtained is valid for 35° onwards
- For head wave interference region to reduce to 35° by decaying, the radius for measuring the directivity is **150mm**